Magnetotransport in a periodically modulated graphene monolayer

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Motivated by the realization that periodic ripples in suspended graphene can result in a periodic potential, which can also be induced by interaction with a substrate, we have carried out a detailed investigation of the electrical transport properties of a graphene monolayer which is modulated by a weak one-dimensional periodic potential in the presence of a perpendicular magnetic field (B). The periodic modulation broadens the Landau Levels into bands which oscillate with B. The electronic conduction in this system can take place through either diffusive scattering or collisional scattering off impurities. Both these contributions to electronic transport are taken into account in this work. In addition to the appearance of commensurability oscillations in both the collisional and diffusive contributions, we find that Hall resistance also exhibits commensurability oscillations. Furthermore, the period and amplitude of these commensurability oscillations in the transport parameters and the effect of temperature on them are also discussed in this work.

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I. INTRODUCTION

Recent successful preparation of a single layer of graphene has generated a lot of interest in this system as experimental and theoretical studies have shown that the nature of quasiparticles in this two-dimensional system is very different from those of conventional two-dimensional electron gas (2DEG) systems realized in semiconductor heterostructures. Graphene has a honeycomb lattice of carbon atoms. The quasiparticles in graphene have a band structure in which electron and hole bands touch at two points in the Brillouin zone. At these Dirac points the quasiparticles obey the massless Dirac equation. In other words, they behave as massless, chiral Dirac Fermions leading to a linear dispersion relation $E_k=\hbar v_F k$ (with the characteristic velocity $v_F \simeq 10^6$ m/s). This difference in the nature of the quasiparticles in graphene from a conventional 2DEG has given rise to a host of unusual phenomena such as the anomalous quantum Hall effect[1,2] with profound effects on transport in these systems. The transport properties of graphene are currently being explored in the presence of nonuniform potentials, such as in $p-n$ junctions,[3] as well as in periodic potentials. Effects of periodic potential on electron transport in 2D electron systems has been the subject of continued interest, where electrical modulation of the 2D system can be carried out by depositing an array of parallel metallic strips on the surface or through two interfering laser beams.[4] More recently in graphene, electrostatic,[5] and magnetic[6] periodic potentials have been shown to modulate its electronic structure in unique ways leading to fascinating physics and possible applications. Periodic potentials are induced in graphene by interaction with a substrate[7] or controlled adatom deposition.[8] In this context, it was recently shown,[7] epitaxial graphene on Ir(111) substrate induces a weak periodic potential in graphene. In addition, it was shown that periodic ripples in suspended graphene also induces a periodic potential in a perpendicular electric field.[9] Epitaxial growth of graphene on top of a prepatterned substrate is also a possible route to modulation of the potential seen by the electrons. In this work, we complement these recent studies to discuss the effects of a weak electric modulation on the electrical conductivity in a graphene monolayer subjected to an external magnetic field perpendicular to the graphene plane. Electric modulation introduces a length scale, period of modulation, in the system giving rise to interesting physical effects on the transport response. Commensurability (Weiss) oscillations, in addition to Shubnikov de Hass (SdH) oscillations, are found to occur as a result of commensurability of the electron cyclotron diameter at the Fermi energy and the period of the electric modulation. In Ref. 10, on the same subject, diffusive contribution to magnetoconductivity in the presence of modulation was considered. There, contribution of Weiss oscillations to magnetoconductivity was presented without taking into account SdH oscillations. In the present work, we determine collisional and Hall contributions along with diffusive contributions to conductivity. We take into account not only Weiss oscillations but also SdH oscillations. As for the relative contribution of diffusive and collisional conductivity in magnetotransport in this system, it primarily depends on the amount of impurity scattering in the system through the impurity concentration, N_I. For realistic values of N_I, the collisional contribution to conductivity becomes significant as we show in this work. This makes this paper a complete study of electric modulation induced effects on electrical conductivity/resistivity in a graphene monolayer in the presence of a magnetic field.

In the next section, we present the formulation of the problem and derive expressions for electrical conductivities in a graphene monolayer. In Sec. III, results of numerical work are presented and discussed, followed by the conclusions.

II. FORMULATION

We consider a graphene sheet in the $x-y$ plane. The magnetic field **B** is applied along the z direction. The system is

also subjected to one-dimensional (1D) weak periodic modulation $U(x)$ in the x direction. The one-electron Hamiltonian reads

$$H = v_F \boldsymbol{\sigma} . (\mathbf{p} + e\mathbf{A}) + U(x), \tag{1}$$

where p is the momentum operator, $\sigma=\{\sigma_x,\sigma_y\}$ are Pauli matrices, and $v_F(\sim 10^6$ m/s) characterizes the electron velocity in graphene. In the absence of modulation, i.e., for $U(x)=0$ and for the vector potential chosen in the Landau gauge $A=(0,Bx,0)$, the normalized eigenfunctions of Eq. (1) are given by $\frac{e^{ik_y y}}{\sqrt{2L_y l}}\begin{pmatrix} -i\phi_n(\frac{x+x_o}{l}) \\ \phi_{n-1}(\frac{x+x_o}{l}) \end{pmatrix}$ where $\phi_n(x)$ and $\phi_{n-1}(x)$ are the harmonic oscillator wave functions centered at $x_o=l^2k_y$. n is the Landau-level index, $l=\sqrt{\frac{\hbar}{eB}}$ the magnetic length and L_y the length of 2D graphene system in the y direction. The corresponding eigenvalue is $E_n=\hbar\omega_g\sqrt{n}$ where $\omega_g=v_F\sqrt{2eB/\hbar}=v_F\sqrt{2}/l$.

The modulation potential is approximated by the first Fourier component of the periodic potential $U(x)=V_o\cos Kx$ where $K=2\pi/a$, a is the period of modulation and V_o is the constant modulation amplitude. This potential lifts the degeneracy of Landau levels (LLs) and the energy becomes dependent on the position x_o of the guiding center. Thus energy eigenvalues for weak modulation ($V_o \ll E_F$), using first-order perturbation theory, are

$$E_{n,k_y} = E_n + V_{n,B}\cos Kx_o, \tag{2}$$

where $V_{n,B}=\frac{V_o}{2}e^{-u/2}[L_n(u)+L_{n-1}(u)]$ with $L_n(u)$ and $L_{n-1}(u)$ the Laguerre polynomials and $u=K^2l^2/2$. We note that the electric modulation induced broadening of the energy spectrum is nonuniform. The Landau bandwidth $\sim V_{n,B}$ oscillates as a function of n since $L_n(u)$ are oscillatory functions of index n. $V_{n,B}$ at the Fermi energy can be approximated, using an asymptotic expression for $n \gg 1$ appropriate for low magnetic-field range relevant to the present study, as

$$V_B = V_o\sqrt{\frac{2}{\pi KR_c}}\cos\left(KR_c - \frac{\pi}{4}\right) \tag{3}$$

where $R_c=k_Fl^2$ is the classical cyclotron orbit, $k_F=\sqrt{2\pi n_e}$ and n_e is the electron number density. The above expression shows that V_B oscillates with B, through R_c, and the width of Landau bands $2|V_B|$ becomes maximum at

$$\frac{2R_c}{a} = i + \frac{1}{4} \quad (i = 1,2,3,\ldots), \tag{4}$$

and vanishes at

$$\frac{2R_c}{a} = i - \frac{1}{4} \quad (i = 1,2,3,\ldots), \tag{5}$$

which is termed the flat band condition. In Fig. 1, half Landau bandwidth at the Fermi energy as a function of the magnetic field is shown. The oscillations of the Landau bandwidth is the origin of the commensurability (Weiss) oscillations and, at the same time, are responsible for the modulation of the amplitude and the phase of the SdH oscillations.

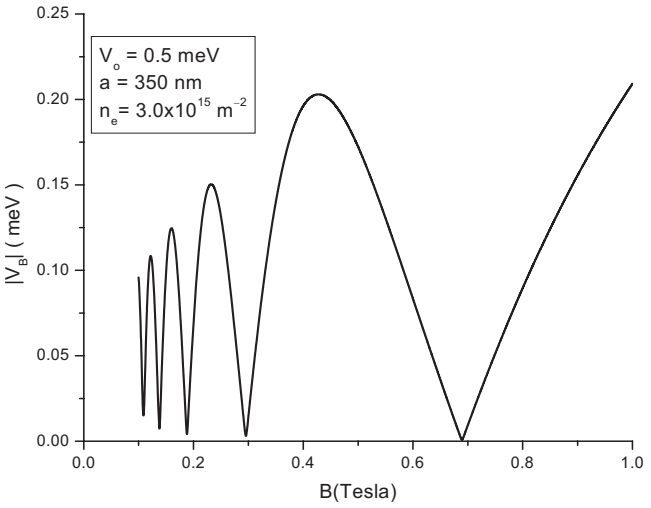

FIG. 1. The half bandwidth of the Landau level at Fermi energy in graphene as a function of magnetic field.

In the presence of a periodic modulation, there are two contributions to magnetoconductivity: the collisional (hopping) contribution and the diffusive (band) contribution. The former is the localized state contribution which carries the effects of SdH oscillations that are modified by periodic modulation. The diffusive contribution is the extended state contribution and arises due to finite drift velocity acquired by the charge carriers in the presence of modulation. To calculate the electrical conductivity in the presence of weak modulation we will follow the formulation of Refs. 4 and 11, which is derived from general Liouville equation and includes dissipative effects. In the linear response regime, the conductivity tensor is a sum of a diagonal and a nondiagonal part: $\sigma_{\mu\nu}(\omega)=\sigma^d_{\mu\nu}(\omega)+\sigma^{nd}_{\mu\nu}(\omega)$ and $\mu,\nu=x,y$. In general, the diagonal conductivity $\sigma^d_{\mu\nu}(\omega)=\sigma^{diff}_{\mu\nu}(\omega)+\sigma^{\rm col}_{\mu\nu}(\omega)$, accounts for both diffusive and collisional contributions whereas the Hall contribution is obtained from nondiagonal conductivity $\sigma^{nd}_{\mu\nu}(\omega)$. Here, $\sigma_{xx}=\sigma^{\rm col}_{xx}$ and $\sigma_{yy}=\sigma^{\rm col}_{xx}+\sigma^{diff}_{yy}$. This formulation has been employed successfully in electronic transport in 2DEG systems and more recently in graphene,[4,11,12] and references therein. The zero frequency (dc) diffusive conductivity has already been determined for a graphene monolayer in Ref. 10. Our focus, in this work, will be the calculation of the dc collisional contribution to the conductivity and the Hall conductivity.

A. Collisional conductivity

To obtain collisional contribution to conductivity, we assume that electrons are elastically scattered by randomly distributed charged impurities as it has been shown that charged impurities play a key role in the transport properties of graphene near the Dirac point.[13,14] This type of scattering is dominant at low temperature. The collisional conductivity when spin degeneracy is considered is given by[4,11]

$$\sigma_{xx}^{\mathrm{col}} = \frac{\beta e^2}{\Omega} \sum_{\xi,\xi'} f_\xi (1 - f_{\xi'}) W_{\xi\xi'} (\alpha_x^\xi - \alpha_x^{\xi'})^2, \tag{6}$$

where $f_\xi = [\exp(\frac{E_\xi - \mu}{k_B T} + 1)]^{-1}$ is the Fermi Dirac distribution function with $f_\xi = f_{\xi'}$ for elastic scattering, k_B is the Boltzmann constant, and μ the chemical potential. $W_{\xi\xi'}$ is the transmission rate between the one-electron states $|\xi\rangle$ and $|\xi'\rangle$, Ω the volume of the system, e the electron charge, $\tau(E)$ the relaxation time, and $\alpha_x^\xi = \langle \xi | r_x | \xi \rangle$ the mean value of the x component of the position operator when the electron is in state $|\xi\rangle$.

Collisional conductivity arises as a result of migration of the cyclotron orbit due to scattering by charge impurities. The scattering rate $W_{\xi\xi'}$ is given by

$$W_{\xi\xi'} = \sum_q |U_q|^2 |\langle \xi | e^{iq.(r-R)} | \xi' \rangle|^2 \delta(E_\xi - E_{\xi'}). \tag{7}$$

The Fourier transform of the screened impurity potential is $U_q = 2\pi e^2 / \varepsilon \sqrt{q^2 + k_s^2}$, where r and R are the position of electron and of impurity, respectively; k_s is the screening wave vector, ε is the dielectric constant of the material. By performing an average over random distribution of impurities, ($N_I \equiv$ impurity density), the contribution of the unperturbed part of the wave function, $|\xi\rangle \equiv |n, k_y\rangle$, to the scattering rate is

$$W_{\xi\xi'}^{(\circ)} = \frac{2\pi N_I}{A_\circ \hbar} \sum_q |U_q|^2 |\langle n, k_y | e^{iq.(r-R)} | n', k_y' \rangle|^2 \delta(E_{n,k_y} - E_{n',k_y'}), \tag{8}$$

with

$$|\langle n, k_y | e^{iq.(r-R)} | n', k_y' \rangle|^2 = \frac{1}{4} [J_{n,n'}(\gamma) + J_{n-1,n'-1}(\gamma)] \delta_{k_y - k_y', q_y}, \tag{9}$$

and

$$|J_{n,n'}(\gamma)|^2 = \frac{n!}{n'!} e^{-\gamma} \gamma^{n-n'} [L_{n'}^{n-1}(\gamma)]^2; \quad n' \le n. \tag{10}$$

Here $A_\circ = L_x L_y$ is the area of the graphene monolayer and $\gamma = l^2 (q_x^2 + q_y^2)/2 = \frac{q_\perp^2 l^2}{2}$ with $q_\perp^2 = (q_x^2 + q_y^2)$. Inserting Eq. (8) in Eq. (6) we obtain

$$\sigma_{xx}^{\mathrm{col}} = \frac{e^2 \beta l^4}{A_\circ} \frac{2\pi N_I}{A_\circ \hbar} \sum_{n,k_y} \sum_{n',k_y} \sum_q |U_q|^2 \frac{1}{4} [J_{n,n'}(\gamma) + J_{n-1,n'-1}(\gamma)]^2 q_y f_{n,k_y} (1 - f_{n,k_y}) \delta(E_{n,k_y} - E_{n',k_y}), \tag{11}$$

with $f_{n,k_y} \equiv f(E_{n,k_y})$, the Fermi Dirac distribution function. Taking $\Sigma_q \to \frac{A_\circ}{4\pi^2 l^2} \int_0^{2\pi} d\varphi \int_0^\infty d\gamma$ and $q_y = q_\perp \sin\varphi$, $|U_q|^2 \sim |U_\circ|^2$ in Eq. (11), we obtain

$$\sigma_{xx}^{\mathrm{col}} = \frac{e^2 \beta N_I}{A_\circ \hbar} |U_\circ|^2 \sum_{n,n',k_y} f_{n,k_y} (1 - f_{n,k_y}) \int_0^\infty \frac{1}{4} \gamma [J_{n,n'}(\gamma) + J_{n-1,n'-1}(\gamma)]^2 d\gamma \delta(E_{n,k_y} - E_{n',k_y}). \tag{12}$$

Using the following integral identity:[4,11,15]

$$\int_0^\infty \gamma [J_{n,n'}(\gamma)]^2 d\gamma = \int_0^\infty \gamma e^{-\gamma} [L_n(\gamma)]^2 d\gamma = (2n+1), \tag{13}$$

where for $n = n'$ and $[J_{n,n'}(\gamma)]^2 = e^{-\gamma} [L_n(\gamma)]^2$ with the result

$$\int_0^\infty \gamma [J_{n-1.n'-1}(\gamma)]^2 d\gamma = \int_0^\infty \gamma e^{-\gamma} [L_{n-1}(\gamma)]^2 d\gamma = (2n-1), \tag{14}$$

$$\int_0^\infty \gamma J_{n,n'}(\gamma) J_{n-1,n'-1}(\gamma) d\gamma = \int_0^\infty \gamma e^{-\gamma} [L_n(\gamma)][L_{n-1}(\gamma)] d\gamma = 0. \tag{15}$$

Finally, replacing the δ function by a Lorentzian of zero shift and constant width Γ, $\Sigma_{k_y} \to \frac{L_y}{2\pi} \int_0^{a/l^2} dk_y$, $A_\circ \to L_x L_y$, and performing the sum on n', keeping only the dominant term $n' = n$ in Eq. (12), we obtain the following result

$$\sigma_{xx}^{\mathrm{col}} \approx \frac{e^2}{h} \frac{N_I U_\circ^2}{\pi a \Gamma} \sum_{n=0}^\infty n \int_0^{a/l^2} dk_y \beta f_{n,k_y} (1 - f_{n,k_y}). \tag{16}$$

B. Diffusive conductivity

For completeness, we also present the result for diffusive conductivity which was determined in,[10]

$$\sigma_{yy}^{diff} = 2\pi^2 \frac{e^2}{h} \frac{V_o^2 \tau}{\hbar} u e^{-u} \sum_{n=0}^\infty \left[\frac{-\partial f(E)}{\partial E} \right]_{E=E_n} [L_n(u) + L_{n-1}(u)]^2, \tag{17}$$

where τ is the scattering time and $\frac{-\partial f(E)}{\partial E} = \beta \exp \beta(E - E_F) / [\exp \beta(E - E_F) + 1]^2$. Here, we have taken the scattering time to be independent of energy and Landau-level index n, which for weak magnetic fields is a reasonable approximation. It is given by $\tau = \frac{\mu E_F}{e v_F^2}$ where μ is the mobility of graphene at zero magnetic field and it is taken to be ~ 4 m^2/V s.[2,14,16]

C. Hall conductivity

The Hall conductivity σ_{yx} is obtained from the nondiagonal elements of the conductivity tensor, as shown in Refs. 4 and 11. This is the diffusive contribution as the collisional contribution to Hall conductivity vanishes, since the difference of the matrix elements $\alpha_y^\xi - \alpha_y^{\xi'} = 0$, where $\alpha_y^\xi = \langle \xi | r_y | \xi \rangle$ is the mean value of the y component of the position operator when the electron is in state $|\xi\rangle$. Hence, σ_{yx} is given by

$$\sigma_{yx} = \frac{2i\hbar e^2}{\Omega} \sum_{\xi \neq \xi'} f_\xi (1 - f_{\xi'}) \langle \xi | v_y | \xi' \rangle \langle \xi' | v_x | \xi \rangle \frac{1 - e^{\beta(E_\xi - E_{\xi'})}}{(E_\xi - E_{\xi'})^2}. \tag{18}$$

Since $f_\xi(1-f_{\xi'})(1-e^{\beta(E_\xi - E_{\xi'})}) = f_{\xi'}(1-f_\xi)$ and $\Omega \to A_\circ \equiv L_x L_y$, we obtain

$$\sigma_{yx} = \frac{2i\hbar e^2}{\Omega} \sum_{\xi \neq \xi'} f_{\xi'} (1 - f_\xi) \frac{\langle \xi | v_y | \xi' \rangle \langle \xi' | v_x | \xi \rangle}{(E_\xi - E_{\xi'})^2}. \tag{19}$$

Since the x and y components of velocity operator are $v_x = \frac{\partial H_\circ}{\partial p_x}$ and $v_y = \frac{\partial H_\circ}{\partial p_y}$ when $H_\circ = v_F \sigma.(p + eA)$. Therefore, $v_x = v_F \sigma_x$ and $v_y = v_F \sigma_y$. Hence,

$$\langle \xi' | v_x | \xi \rangle = \langle n', k_y | v_x | n, k_y \rangle = -i v_F, \tag{20}$$

and

$$\langle \xi | v_y | \xi' \rangle = \langle n, k_y | v_y | n', k_y \rangle = v_F. \tag{21}$$

Substituting the values of the matrix elements of velocity in Eq. (19) yields

$$\sigma_{yx} = \frac{2\hbar e^2 v_F^2}{L_x L y} \sum_{\xi \neq \xi'} \frac{f_{\xi'}(1 - f_\xi)}{(E_\xi - E_{\xi'})^2}. \tag{22}$$

Since $E_\xi \equiv E_{n,k_y} = E_n + V_{n,B} \cos K x_o$ where $E_n = \hbar \omega_g \sqrt{n}$ and $V_{n,B} = \frac{V_o}{2} e^{-u/2} [L_n(u) + L_{n-1}(u)]$ we obtain

$$(E_\xi - E_{\xi'})^2 = \hbar^2 \omega_g^2 [\sqrt{n+1} - \sqrt{n} + \lambda_n \cos K x_\circ]^2, \tag{23}$$

where

$$\lambda_n = \frac{V_\circ}{2\hbar \omega_g} e^{-u/2} [L_{n+1}(u) - L_{n-1}(u)]. \tag{24}$$

Substituting Eq. (23) in Eq. (22) we obtain the Hall conductivity in graphene as

$$\sigma_{yx} = \frac{e^2}{h} \frac{l^2}{a} \sum_{n=0}^{\infty} \int_0^{a/l^2} dk_y \frac{f_{n,k_y} - f_{n+1,k_y}}{[\sqrt{n+1} - \sqrt{n} + \lambda_n \cos K x_\circ]^2}. \tag{25}$$

Elements of the resistivity tensor $\rho_{\mu\nu}(\mu, \nu = x, y)$ can be determined from those of the conductivity tensor $\sigma_{\mu\nu}$, obtained above, using the expressions: $\rho_{xx} = \sigma_{yy}/S$, $\rho_{yy} = \sigma_{xx}/S$, and $\rho_{xy} = \sigma_{yx}/S$ where $S = \sigma_{xx}\sigma_{yy} - \sigma_{xy}\sigma_{yx}$ with $S \approx \sigma_{xy}^2 = n_e^2 e^2 / B^2$.

III. RESULTS AND DISCUSSION

The above expressions for the (collisional, diffusive, and Hall) conductivities, Eqs. (16), (17), and (25) are the principal results of this work. The integrals appearing in these equations are evaluated numerically and the results are presented in Fig. 2(a) at temperature $T=2$ K for a graphene monolayer with electron density $n_e = 3.0 \times 10^{11}$ cm^{-2}, electric modulation strength $V_o = 0.5$ meV with period $a = 350$ nm. For the aforementioned value of n_e, the Fermi energy of the system $E_F = \hbar v_F \sqrt{2\pi n_e} \approx 90.5$ meV. In addition, the following parameters were employed:[2,14,16] $\tau = 4 \times 10^{-13}$ s, $\Gamma = 0.4$ meV, impurity density $N_I = 2.5 \times 10^{11}$ cm^{-2}, and $\varepsilon = 3.9$ (using SiO_2 as the substrate material). We observe that SdH oscillations are visible in collisional conductivity σ_{xx}. Furthermore, Weiss oscillations superimposed on SdH oscillations are seen in σ_{yy}. To highlight the effects of modulation, we also calculate the correction to the conductivity (change in conductivity) as a result of modulation which is expressed as $\Delta\sigma_{\mu\nu} = \sigma_{\mu\nu}(V_o) - \sigma_{\mu\nu}(V_o = 0)$ and is shown in Fig. 2(b). Electric modulation acting on the system results in a positive contribution to $\Delta\sigma_{yy}$ and a negative contribution to $\Delta\sigma_{xx}$ whereas, $\Delta\sigma_{yx}$ oscillates around zero. We find that $\Delta\sigma_{yy} \gg \Delta\sigma_{xx}$, which is a consequence of the fact that $\Delta\sigma_{xx}$ has only collisional contribution, while $\Delta\sigma_{yy}$, in addition to the collisional part, has contributions due to band conduction which are much larger. It is also seen that the oscillations in $\Delta\sigma_{xx}$ and $\Delta\sigma_{yy}$ are 180° out of phase. To determine the effects of temperature on magnetoconductivities, comparison of conductivities and corrections to the conductivities at two different temperatures $T=2$ K (solid curve) and $T=6$ K (broken curve) are presented in Figs. 3 and 4 respectively. $\Delta\sigma_{xx}$ shows strong temperature dependence which is a clear signature that SdH oscillations are dominant here. Oscillations in $\Delta\sigma_{yy}$ show comparatively weaker dependence on temperature as Weiss oscillations, that are weakly dependent on temperature, play a more significant role in σ_{yy}. Furthermore, Weiss oscillations are also seen in $\Delta\sigma_{yx}$ and they are weakly sensitive to temperature a

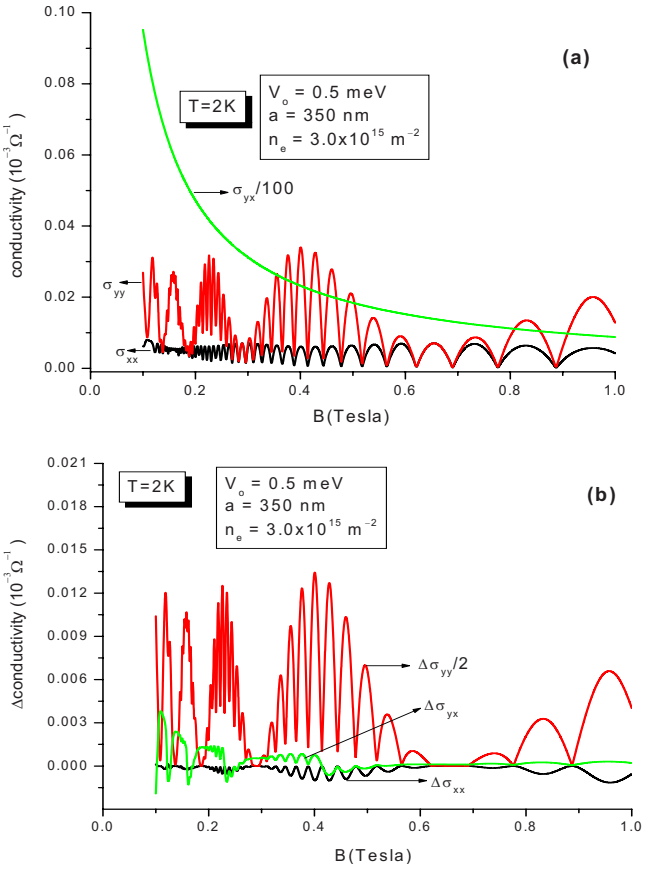

FIG. 2. (Color online) The components of the conductivity tensor of graphene (a) and change in conductivity due to the 1D modulation (b) as function of magnetic field at temperature (T=2 K).

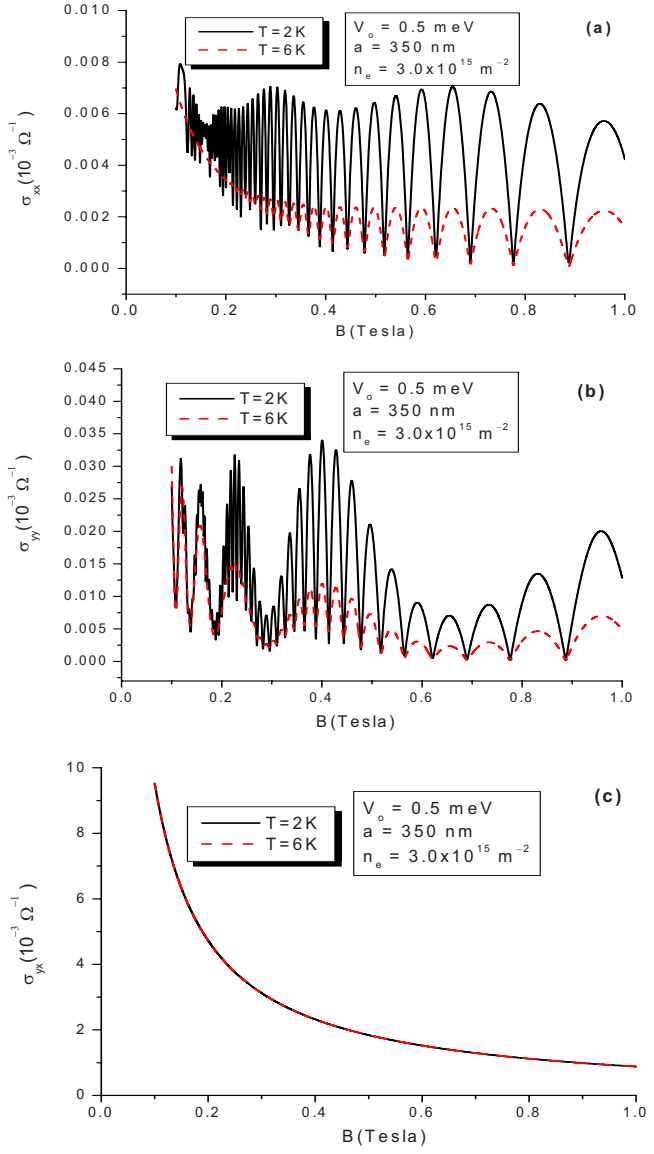

FIG. 3. (Color online) Conductivities [(a) collisional; (b) diffusive+collisional, (c) Hall] of graphene as function of magnetic field at two different temperatures (T=2 K; solid curve and T =6 K; broken curve). The 1D modulation is along the T=6 K direction.

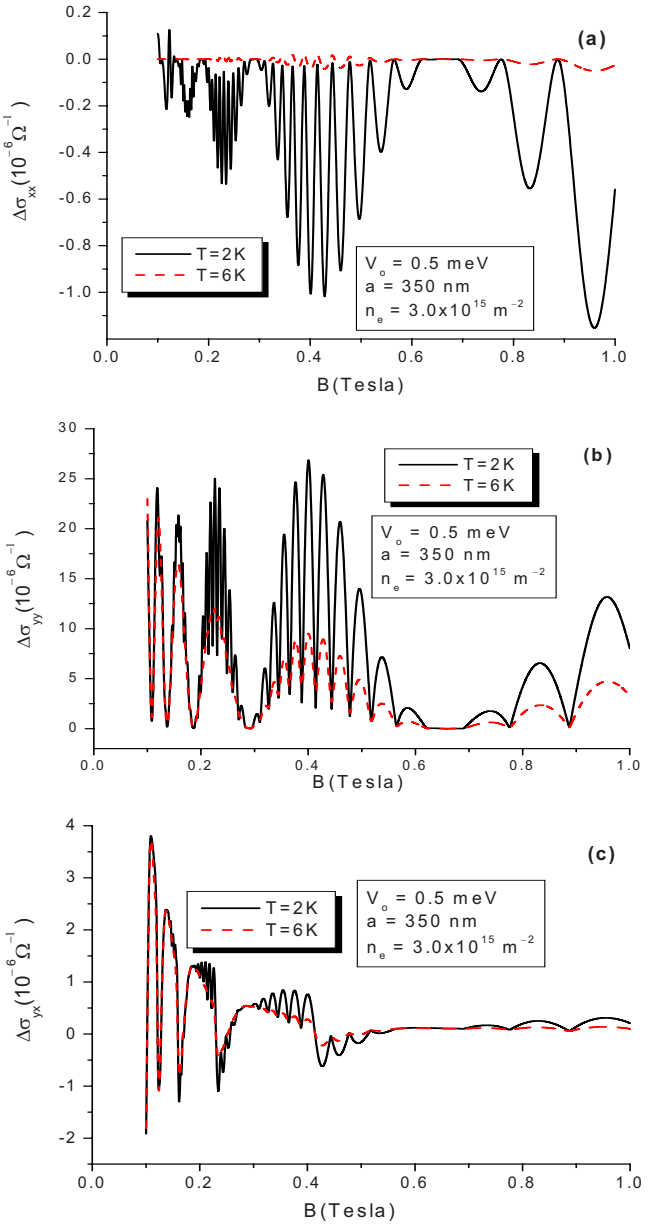

FIG. 4. (Color online) Corrections to the conductivities [(a) collisional; (b) diffusive+collisional, and (c) Hall] of graphene as function of magnetic field at two different temperatures (T=2 K; solid curve and T=6 K; broken curve). The 1D modulation is along the x direction.

low magnetic fields (that is when $B<0.188$ T). In graphene system, the value of B defining the boundary between SdH and Weiss oscillations is quite low (it lies between 0.1 and 0.15 Tesla). For smaller values of B, the amplitude of Weiss oscillations remain essentially the same at various temperatures. When B is large, SdH oscillations dominate and the amplitude of oscillations gets reduced considerably at comparatively higher temperatures. However, oscillatory phenomenon still persists.

It can be seen from Figs. 2(a), 3(a), and 3(b) that amplitude of SdH oscillations remains large at those values of the magnetic field where the flat band condition is satisfied i.e., at $B(\text{Tesla})=0.6897, 0.2956, 0.1881, 0.1379, 0.1089\ldots$ when $i=1,2,3,4\ldots$ in Eq. (5) while is suppressed at the maximum bandwidth/broad band condition, i.e., at $B(\text{Tesla})=0.4138, 0.2299, 0.1592, 0.1217, 0.0985, \ldots$ for $i=1,2,3,4,\ldots$ in Eq. (4). These values agree with the position of the minima and maxima in the bandwidth shown in Fig. 1. Furthermore, zeros in $\Delta\sigma_{\mu\nu}$ appear in close agreement with values predicted from the flat band condition. The amplitude of $\Delta\sigma_{xx}$ and $\Delta\sigma_{yy}$ becomes maximum at the broad band condition [as seen in Fig. 4], whereas the amplitude of $\Delta\sigma_{yx}$ crosses the zero level at the broad band condition and than a phase change in amplitude occurs.

Components of the resistivity tensor $\rho_{\mu\nu}$ have also been computed and shown in Figs. 5(a) and 6 as a function of B for $T=2$ K (solid curve) and 6 K (broken curve), respectively. The correction (change) in $\rho_{\mu\nu}$ due to the modulation

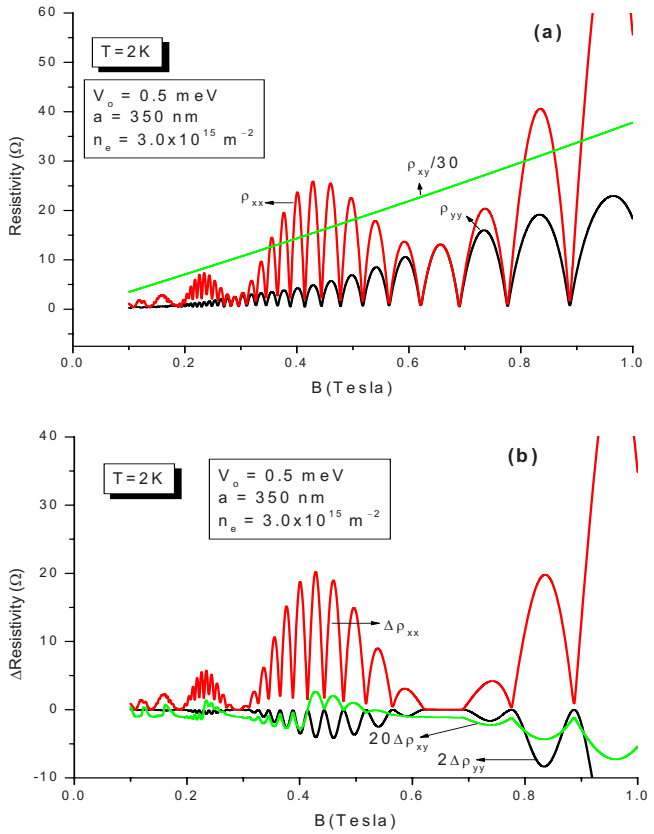

FIG. 5. (Color online) The components of the resistivity tensor of graphene (a) and change in resistivity due to the 1D modulation (b) as function of magnetic field at temperature (T=2 K).

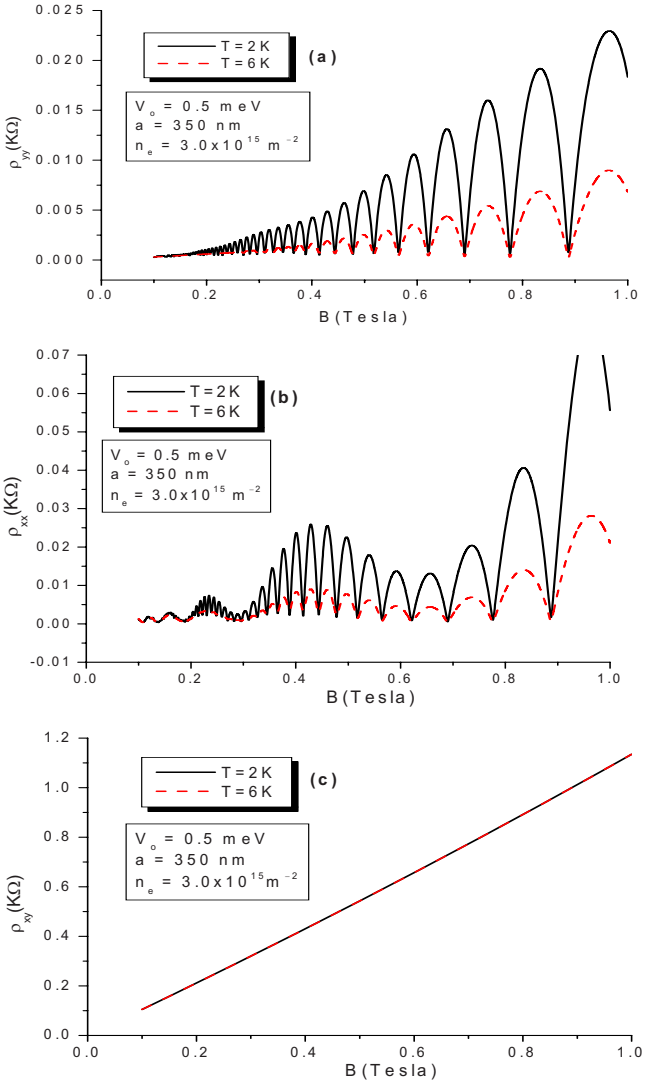

FIG. 6. (Color online) Resistivities [(a) collisional; (b) diffusive+collisional, and (c) Hall] of graphene as function of magnetic field at two different temperatures (T=2 K; solid curve and T=6 K; broken curve). The 1D modulation is along the x direction.

is shown in Figs. 5(b) and 7. To verify our results, we compare them in the absence of modulation with the unmodulated experimental results presented in Ref. 17. In order to carry this out, we note that the number density n_e is related to the gate voltage (V_g) through the relationship[18] $n_e = \epsilon_o \epsilon V_g / te$, where ϵ_o and ϵ are the permittivities of free space and ϵ=3.9 (graphene on SiO_2 substrate), respectively. e is the electron charge and $t(\approx 350$ nm) the thickness of the sample. It yields V_g=4.8 V for $n_e = 3.0 \times 10^{11}$ cm^{-2}. We find that the qualitative results for magnetoresistivities obtained in this work are in good agreement with the values given in Ref. 17 for the unmodulated case at V_g=4.8 V.

We observe in Fig. 5(a), that the dominant effect of Weiss oscillations appears in ρ_{xx} as it is proportional to σ_{yy} whereas the amplitude of oscillations in ρ_{yy} show a monotonic increase in amplitude with magnetic field signifying dominance of SdH in ρ_{yy}. In Fig. 5(b), we observe that the oscillations in $\Delta\rho_{xx}$ and $\Delta\rho_{yy}$ are out of phase and the amplitude of the oscillation in $\Delta\rho_{xx}$ is greater than the amplitude of oscillation in $\Delta\rho_{yy}$. The out of phase character of the oscillations can be understood by realizing that the conduction along the modulation direction, which contributes to ρ_{yy}, occurs due to hopping between Landau states and it is minimum when the density of states at the Fermi level is minimum. Oscillations in ρ_{xx} are much larger than those in ρ_{yy} as a mechanism of conduction due to modulation contributes to ρ_{xx}. To highlight temperature effects on the modulated system, we present in Fig. 7, corrections to magnetoresistivities at two different temperatures (2 K, solid curve and 6 K, broken curve). These results exhibit SdH oscillation when B becomes greater than $0.188T$ as seen in Figs. 6 and 7, respectively. The Weiss oscillations in $\Delta\rho_{xx}$ are in phase with those of $\Delta\rho_{xy}$. From Figs. 3(c) and 6(c) one might infer that Hall conductivity and hence resistivity is not affected by modulation. This is not so, as Hall conductivity/resistivity carry modulation effects and that is seen if we consider the slope of ρ_{xy} as a function of magnetic field [Fig. 8]. In the absence of modulation λ_n=0 and $f_{n,k_y} \equiv f_n$; and strong magnetic fields Eq. (25) leads to $\sigma_{xy} = 4n\frac{e^2}{h}$ (integral quantum Hall effect in graphene) with the assumption that the Fermi level lies in the region of localized states between two successive Landau levels. However, in the presence of modulation and in a weak magnetic field $\lambda_n \neq 0$, the term $\hbar\omega[\sqrt{n+1}-\sqrt{n} + \lambda_n \cos Kx_\circ]$ in Eq. (25) expressing the energy difference $E_\xi - E_{\xi'}$ between successive Landau levels oscillates with magnetic field and leads to oscillations in σ_{yx} and ρ_{xy}, re-

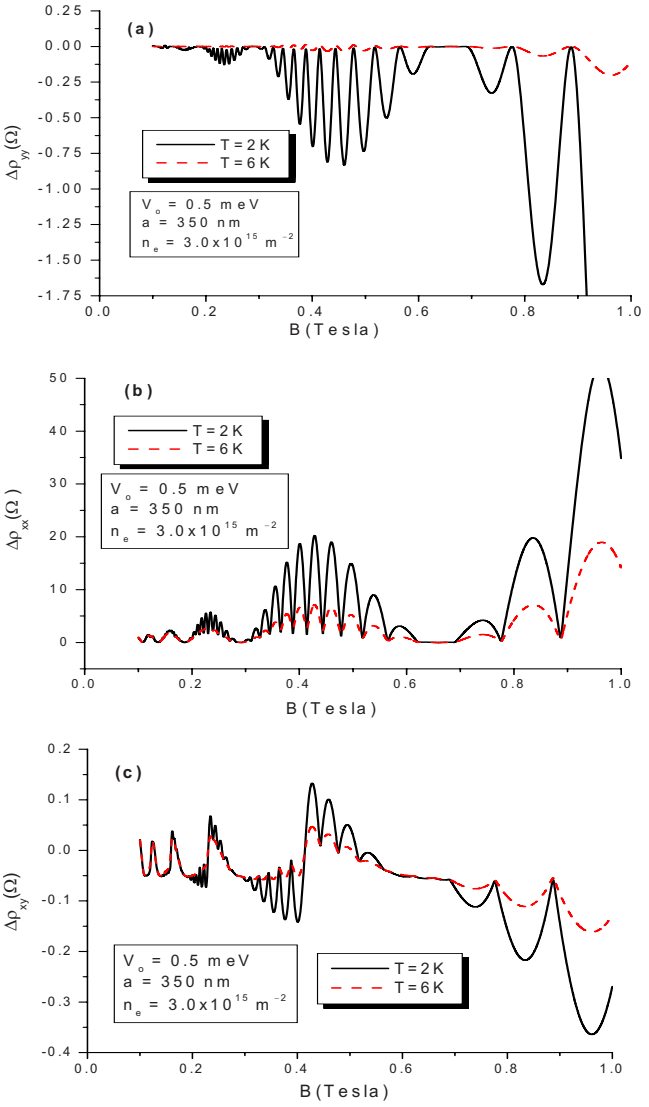

FIG. 7. (Color online) Corrections to the resistivities [(a) collisional; (b) diffusive+collisional, (c) Hall] of graphene due to the periodic 1D modulation potential at two different temperatures (T =2 K; solid curve and T=6 K; broken curve).

spectively. These oscillations are small as they result from the term $\lambda_n \cos Kx_\circ$ which is the difference of the bandwidths of the two neighboring Landau levels. If we take the derivative of Hall resistance with respect to magnetic field, as shown in Fig. 8, these oscillations can be observed.

In order to quantitatively analyze the results presented in the figures we consider the density of states (DOS) of this system. At finite temperature, the oscillatory part of resistivities $(\Delta\rho/\rho_o)$ are proportional to the oscillatory part of the DOS at the Fermi energy, $A(T/T_c)\Delta D(E_F)/D_o$ where $A(T/T_c)=(\frac{T}{T_c})/\sinh(\frac{T}{T_c})$, D_o is the DOS and ρ_o is resistivity in the absence of magnetic field, respectively.[12] For not too small magnetic fields $(B \gtrsim 0.05T)$ and $\Delta\rho/\rho_o \simeq (\omega_g\tau)^2\Delta\sigma/\sigma_o$ to a good approximation, where $\sigma_o = \frac{e^2 v_F^2}{2}\tau D_o$ represents conductivity at zero magnetic field and τ is the relaxation time. The analytic expression for the DOS of a graphene monolayer in the presence of a magnetic field

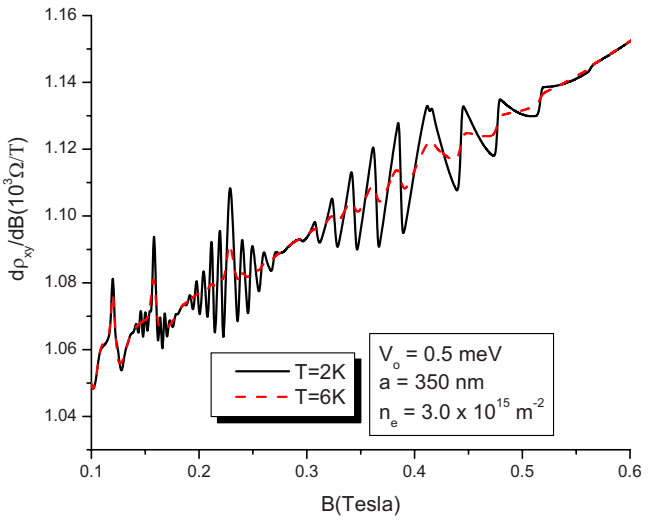

FIG. 8. (Color online) The derivative of the Hall resistance with respect to the magnetic field for T=2 K (solid curve) and T =6 K (broken curve).

subjected to electric modulation has been derived in the Appendix. The DOS at energy E is given as

$$D(E,V_B) = D_o\left[1 + 2\sum_{k=1}^{\infty}\frac{1}{2\pi}\int_0^{2\pi}\cos[2\pi k(\varepsilon - v_B\cos t)]dt \times\exp(-2\pi k\eta)\right]$$
$$= D_o\left[1 + 2\sum_{k=1}^{\infty}\cos(2\pi k\varepsilon)J_o(2\pi k v_B)\exp(-2\pi k\eta)\right] \quad (26)$$

where $D_o = \frac{2E}{(\hbar\omega_g)^2\pi l^2} = \frac{E}{\pi(\hbar v_F)^2}$, $\epsilon = (\frac{E}{\hbar\omega_g})^2$, $\eta = \frac{\Gamma E}{(\hbar\omega_g)^2}$, and $v_B = \frac{2V_B E}{(\hbar\omega_g)^2}$. $J_o(x)$ is the Bessel function of order zero. Since $\exp(-2\pi k\eta) \ll 1$ for weak magnetic fields, it is usually a good approximation to keep only the $k=1$ term in the sum: $D(E) \simeq D_o + \Delta D_1(E)$ with

$$\frac{\Delta D_1(E)}{D_o} = 2\cos(2\pi\varepsilon)J_o(2\pi v_B)\exp(-2\pi\eta). \quad (27)$$

To determine the effects of an external magnetic field on the conductivities/resistivities of the system we consider Eq. (26). With a decrease in B, v_B oscillates periodically with respect to $1/B$ around $v_B=0$, increasing its amplitude proportionally to $1/\sqrt{B}$ [Eq. (3)]. The function $J_o(2\pi v_B)$ decreases from 1 with an increase of $|v_B|=0.3827 \simeq 3/8$ and than changes its sign. Therefore the oscillations of $\Delta D_1(E)$ takes a minimum amplitude at the maximum bandwidth conditions while $|v_B|$ stays less than 3/8; it disappears when a maximum of $|v_B|$ touches at $\sim 3/8$; it reappears with an inverted sign for $|v_B|$ larger than 3/8. Therefore, if we assume that $\Delta\rho/[\rho_o A(T/T_c)] \propto \Delta D(E_F)/D_o$ holds, we can find the position where oscillations of $\Delta\rho/[\rho_o A(T/T_c)]$ vanish. That occurs at $|V_B| = 0.19135(\hbar\omega_g)^2/E_F$.

We can also find the period of oscillations in

conductivities/resistivities from Eq. (26) as follows. We have $D(E,V_B)\approx D_o\{1+2\cos(2\pi\varepsilon)J_o(2\pi v_B)\exp(-2\pi k\eta)\}\approx D_o\{1+2\cos(2\pi\varepsilon)(1-\pi^2v_B^2)\exp(-2\pi k\eta)\}$. Since $v_B^2\propto\cos^2(KR_c-\frac{\pi}{4})$. The period of oscillation can be estimated by equating the increment of the cosine argument with π,

$$K\Delta(R_c)=\pi, \tag{28}$$

which leads to

$$\Delta\left(\frac{1}{B}\right)=\left(\frac{e}{2\sqrt{2\pi}\hbar}\right)\frac{a}{\sqrt{n_e}}. \tag{29}$$

In our work ($n_e=3.0\times10^{11}$ cm^{-2} and $a=350$ nm), therefore the period of oscillations comes out to be 1.933 T^{-1} which is in good agreement with the results shown in the figures.

Damping of these oscillations with temperature can also be discussed. In Ref. 10, the temperature scale for damping of Weiss oscillations is given by $K_BT_c^{Weiss}=b\hbar v_F/4\pi^2a$ where $b=(a/l)^2$ and $v_F=\frac{\omega_g l}{\sqrt{2}}$, whence the result

$$K_BT_c^{Weiss}=\frac{a\hbar v_F}{4\pi^2l^2}=\frac{\hbar\omega_g}{2\pi^2}\left(\frac{a}{2\sqrt{2}l}\right). \tag{30}$$

To determine the damping temperature for SdH oscillations we, following Refs. 4, 10, and 11, use asymptotic expression for magnetoconductivity. For this, we use DOS [Eq. (26)],

$$D(E)=\frac{2E}{(\hbar\omega_g)^2\pi l^2}\left[1+2\exp(-2\pi\eta)\cos\left(2\pi\frac{E^2}{(\hbar\omega_g)^2}\right)+\ldots\right]. \tag{31}$$

In the asymptotic limit of weak magnetic fields when many filled Landau levels occur, we take $L_n\approx L_{n-1}$ and replace $e^{-u/2}L_n$ by $1/\sqrt{\pi\sqrt{nu}}\cos(2\sqrt{nu}-\pi/4)$ and inserting the continuum approximation $\Sigma_{n=0}^{\infty}\rightarrow\int_0^{\infty}dED(E)$ in Eq. (17), we obtain the following result

$$\frac{\sigma_{yy}^{diff}}{\sigma_o}=\frac{4\sqrt{2}\pi^2l}{a}\frac{V_o^2}{E_F(\hbar\omega_g)}\left[F+2\exp(-2\pi\eta)A(T/T_c^{SdH})\times\cos\left(2\pi\frac{E_F^2}{(\hbar\omega_g)^2}\right)\cos^2\left(\sqrt{2}Kl\frac{E_F}{\hbar\omega_g}-\frac{\pi}{4}\right)\right] \tag{32}$$

where $F=\frac{1}{2}[1-A(T/T_c^{Weiss})+2A(T/T_c^{Weiss})\cos^2(\sqrt{2}Kl\frac{E_F}{\hbar\omega_g}-\frac{\pi}{4})]$ is the contribution of Weiss oscillations and $A(T/T_c^{SdH})=[4\pi^2E_FK_BT/(\hbar\omega_g)^2]/\sinh[4\pi^2E_FK_BT/(\hbar\omega_g)^2]$ is the amplitude of the SdH oscillations. Therefore, the characteristic temperature of SdH oscillations is given by

$$K_BT_c^{SdH}=\frac{(\hbar\omega_g)^2}{4\pi^2E_F}=\frac{\hbar\omega_g}{2\pi^2}\left(\frac{1}{\sqrt{2}k_Fl}\right). \tag{33}$$

The amplitude of oscillations is given by $A=\frac{x}{\sinh(x)}$, where $x=\frac{T}{T_c}$. The amplitude of Weiss oscillations at $B=0.3$ T are 0.9993 and 0.9938 at $T=2$ and 6 K, respectively. The corresponding amplitudes for SdH oscillations are 0.6878 and 0.0882. The SdH amplitude decreases by ~87 percent whereas the amplitude of Weiss oscillations decreases by ~0.55 percent for 4 K change in temperature. In Figs. 3, 4, 6, and 7; the SdH amplitude decreases by ~77 percent when temperature is changed from $T=2$ to 6 K, and it is in good agreement with the results obtained from Eqs. (30) and (33). It is due to the fact $K_BT_c^{Weiss}\gg K_BT_c^{SdH}$ that the Weiss oscillations are more robust against temperature changes.

Finally, we compare the results obtained for the conductivity/resistivity of graphene with those of a 2DEG given in Refs. 4 and 11. The characteristic damping temperatures for Weiss and SdH oscillations in 2DEG are $K_BT_{2DEG}^{Weiss}=\frac{\hbar\omega_c}{2\pi^2}(\frac{ak_F}{2})$ and $K_BT_{2DEG}^{SdH}=\frac{\hbar\omega_c}{2\pi^2}$, respectively. In contrast, the corresponding damping temperatures in graphene are given by Eq. (30) and (33). On comparing the two temperature scales, we find that the damping temperatures of both oscillations in graphene are higher than that of a 2DEG. The ratio is found to be $\frac{T_c}{T_{c,e}}=\frac{m^*v_F}{\hbar k_F}\approx4.2$; where m^* is the electron mass in a 2DEG and $T_{c,e}$ is the critical temperature of a 2DEG; which implies that a comparatively higher temperature is required for damping of oscillations in graphene. This is due to the higher Fermi velocity of Dirac electrons in graphene compared to standard electrons in a 2DEG systems. It is evident from the numerical results that both, Sdh and Weiss-type oscillations, are more enhanced and more robust against temperature in graphene.

To conclude, in order to study electronic transport in a rippled graphene monolayer or in a graphene monolayer where the substrate induces a periodic potential on the system, we have investigated the effects of a weak periodic electric modulation on the conductivity of a graphene monolayer subjected to a perpendicular magnetic field. As a result of modulation, a length scale, period of modulation, enters the system leading to commensurability oscillations in the diffusive, collisional, and Hall contributions to conductivities/resistivities. These modulation induced effects on graphene magnetotransport are discussed in detail in this work.

APPENDIX

Here we derive the expression for the density of states, Eq. (26) in the text. We consider monolayer graphene subjected to a uniform quantizing magnetic field $\mathbf{B}=B\hat{z}$ in the presence of an additional weak periodic modulation potential. The energy spectrum in the quasi classical approximation, i.e., when many Landau bands are filled may be written as

$$E_{n,x_o}=\sqrt{n}\hbar\omega_g+V_{n,B}\cos Kx_o, \tag{A1}$$

where $V_{n,B}=\frac{V_o}{2}e^{-u/2}[L_n(u)+L_{n-1}(u)]$ with $L_n(u)$ and $L_{n-1}(u)$ the Laguerre polynomials and $u=K^2l^2/2$. For large n; $L_n(u)\approx L_{n-1}(u)$ and $V_{n,B}=V_oe^{-u/2}L_n(u)$. Using the asymptotic expression for the Laguerre polynomials;[15] $e^{-u/2}L_n(u)\rightarrow\frac{1}{\sqrt{\pi\sqrt{nu}}}\cos(2\sqrt{nu}-\frac{\pi}{4})$ and taking the continuum limit $n\rightarrow\frac{1}{2}(\frac{lE}{v_F\hbar})^2$, where $v_F=\omega_gl/\sqrt{2}$ we get

$$V_{n,B}=V_0\pi^{-1/2}\left(\frac{1}{2}K^2l^2\frac{E}{\hbar\omega_g}\right)^{-1/4}\cos\left(\sqrt{2}Kl\frac{E}{\hbar\omega_g}-\frac{\pi}{4}\right). \tag{A2}$$

To obtain a more general result which will lead to the result that we require as a limiting case we consider impurity broadened Landau levels. The self-energy may be expressed as

$$\Sigma^-(E) = \Gamma_o^2 \sum_n \int_0^a \frac{dx_o}{a} \frac{1}{E - E_{n,x_0} - \Sigma^-(E)}, \tag{A3}$$

which yields

$$\Sigma^-(E) = \int_0^a \frac{dx_o}{a} \sum_{-\infty}^{\infty} \frac{\Gamma_o^2}{E - \Sigma^-(E) - V_{n,B} \cos Kx_o - \sqrt{n}\hbar\omega_g}. \tag{A4}$$

Γ_o is the broadening of the levels due to the presence of impurities. The density of states is related to the self energy through

$$D(E) = \mathrm{Im}\left[\frac{\Sigma^-(E)}{\pi^2 l^2 \Gamma_o^2}\right]. \tag{A5}$$

The residue theorem has been used to sum the series $\Sigma_{-\infty}^{\infty} f(n) =$ $-\{$Sum of residues of $\pi(\cot \pi n) f(n)$ at all poles of $f(n)\}$.[19] Here, $f(n) = \Sigma_{-\infty}^{\infty} \frac{b}{c - d\sqrt{n}}$ with $b = \Gamma_o^2$, $c = E - \Sigma^-(E) - V_{n,B} \cos Kx_o$ and $d = \hbar\omega_g$. The function $f(n)$ has a pole at c^2/d^2 and the residue of $[\pi(\cot \pi n) f(n)]$ at the pole is $\frac{-2bc}{d^2} \pi \cot(\frac{\pi c^2}{d^2})$. Hence $\Sigma_{-\infty}^{\infty} f(n) = \frac{2bc}{d^2} \pi \cot(\frac{\pi c^2}{d^2})$ and we obtain

$$\begin{aligned}\Sigma^-(E) &= \int_0^a \frac{dx_o}{a} \frac{2\pi\Gamma_o^2 (E - \Sigma^-(E) - V_{n,B} \cos Kx_o)}{(\hbar\omega_g)^2} \cot\left(\frac{\pi(E - \Sigma^-(E) - V_{n,B} \cos Kx_o)^2}{(\hbar\omega_g)^2}\right) \\ &\approx \frac{2\pi\Gamma_o^2 E}{(\hbar\omega_g)^2} \int_0^a \frac{dx_o}{a} \cot\left(\frac{\pi E}{(\hbar\omega_g)^2}[E - 2\{\Sigma^-(E) + V_{n,B} \cos(Kx_o)\}]\right).\end{aligned} \tag{A6}$$

Separating $\Sigma^-(\varepsilon)$ into real and imaginary parts

$$\Sigma^-(E) = \Delta(E) + i\frac{\Gamma(E)}{2}. \tag{A7}$$

Equation (A6) takes the form

$$\Delta(E) + i\frac{\Gamma(E)}{2} = \frac{2\pi\Gamma_o^2 E}{(\hbar\omega_g)^2} \int_0^a \frac{dx_o}{a} \frac{\sin 2u + i \sinh 2v}{\cosh 2v - \cos 2u}, \tag{A8}$$

where

$$u = \frac{\pi E}{(\hbar\omega_g)^2}[\varepsilon - 2\{\Delta(E) + V_{n,B} \cos(Kx_o)\}], \tag{A9}$$

$$v = \frac{\pi\Gamma(E)E}{(\hbar\omega_g)^2}, \tag{A10}$$

$$\begin{aligned}\mathrm{Im}[\Sigma^-(E)] &= \frac{2\pi\Gamma_o^2 E}{(\hbar\omega_g)^2} \int_0^a \frac{dx_o}{a} \frac{\sinh 2v}{\cosh 2v - \cos 2u} \\ &= \frac{2\pi\Gamma_o^2 E}{(\hbar\omega_g)^2} \int_0^a \frac{dx_o}{a} (1 + 2\sum_{k=1}^{\infty} \cos(2ku) \exp(-2kv).\end{aligned} \tag{A11}$$

If we define dimensionless variables $\varepsilon = (\frac{E}{\hbar\omega_g})^2$, $\eta = \frac{\Gamma E}{(\hbar\omega_g)^2}$, and $v_B = \frac{2V_B E}{(\hbar\omega_g)^2}$ the density of states is obtained as

$$D(E, V_B) = D_o(E)\left\{1 + 2\sum_{k=1}^{\infty} \int_0^a \frac{dx_o}{a} \cos[2\pi k(\varepsilon - v_B \cos K_o x)] \exp(-2\pi k\eta)\right\}, \tag{A12}$$

where $D_o(E) = \frac{2E}{(\hbar\omega_g)^2 \pi l^2}$. Let $Kx_o = t$ in the above expression results in

$$D(E, V_B) = D_o(E)\left\{1 + 2\sum_{k=1}^{\infty} \frac{1}{2\pi} \int_0^{2\pi} \cos[2\pi k(\varepsilon - v_B \cos t)] dt \exp(-2\pi k\eta)\right\}. \tag{A13}$$

Solving the integral yields

$$D(E, V_B) = D_o(E)\left\{1 + 2\sum_{k=1}^{\infty} \cos(2\pi k\varepsilon) J_o(2\pi k v_B) \exp(-2\pi k\eta)\right\}. \tag{A14}$$

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